NASA Technical Memorandum 81346

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December 1979

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INTRODUCTION

In applications where observations are assumed to follow a normal distribution, very often interest centers around the extreme value, since in some sense, such a value indicates the tolerance of a system. The maximum or minimum sample values in such applications are of limited usefulness, because maximum and minimum tend to extend with increasing sample sizes. Moreover, the information actually sought may not pertain to the actual extreme values, but rather may be needed for the values falling above or below some preassigned p-th percentile. In a given application (ref. 1) the information may be needed on the values falling above the 99th percentile. For such an application, the method of extreme mean is presented in this study.

The extreme mean in this study is defined as the mean of a truncated normal distribution above or below a preassigned p-th percentile. An unbiased estimate of this extreme mean is obtained and its variance is then compared with the Cramer-Rao lower bound. Further, the distribution of the standardized estimate and various confidence intervals are obtained.

The distortion parameters data obtained from high frequency response pressure measurements made at the inlet/engine interface plane during a YF-12 flight experiment (ref. 1) are used to demonstrate the usefulness of extreme mean in applications.

SYMBOLS

A	$= (1\sqrt{n}) + af/\sqrt{2(n - 1)}$
a,b	Constants corresponding to extreme mean and variance
B	$= f\sqrt{(2n - 3)/(2n - 2)} - 1$
C	$= \sqrt{(1 + d)/2n(n - 1)}$
c_p	Solution to equation $\Phi[(c_p - \mu)/\sigma] = p$
D	$= \sqrt{2n - 3}$
$D(T)$	Density function of T
d	$= na^2 (f^2 - 1)$
E	Expectation of random variable
$F(T)$	Distribution function of T
f	$= \sqrt{(n - 1)/2}\ \Gamma[(n - 1)/2]/\Gamma(n/2)$
$f(x)$	Density function of the truncated normal distribution
I_{11},I_{12},I_{22}	Elements of information matrix
IDT	Measure of simple distortion
i	Sample number
K_A	Measure of combined circumferential and radial distortion
K_r	Measure of radial distortion
K_Θ	Measure of circumferential distortion
L,U	Lower and upper confidence bounds
l,u	Lower and upper confidence bounds of standardized distribution
$2m$	Length of confidence interval
n	Sample size
P_r	Probability of event
p	Percentile assigned for extreme mean
s	Sample standard deviation

T	$= (\hat{\mu}_E - \mu_E) / \hat{\sigma}_{\mu_E}$, standardized extreme mean variable
$x_1, \ldots, x_n$	Sample values
$\bar{x}$	Sample mean
Y_r	Chi Square random variable with r degrees of freedom
Z	Standard normal random variable
α, ALPHA	Confidence level
$\Gamma(x)$	Gamma function
μ, MU	Mean of normal distribution
μ_E	Extreme mean
σ, SIGMA	Standard deviation of normal distribution
σ_E	Standard deviation of truncated normal distribution
$\hat{}$, HAT	Estimate of associated μ or σ
$\Phi(x)$	Standard normal distribution function

FORMULAE FOR EXTREME MEAN μ_E AND ITS UNBIASED ESTIMATE

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a normal distribution with unknown population parameters, mean μ and standard deviation σ, shown in figure 1. The extreme mean μ_E is defined as the mean of the truncated distribution shown in figure 1, truncated at c_p depending on the preassigned value of p. The sample provides a sample mean, $\bar{x}$, and a sample standard deviation, s, which are sufficient statistics (ref. 2) to estimate any function of μ and σ. These statistics are independent and are employed in the estimation of μ_E and its distribution.

Truncated Distribution Mean μ_E and Variance σ_E^2

The density function of a truncated normal distribution is

$$f(x) = \left[(1 - p)\sigma\sqrt{2\pi}\right]^{-1}\exp\left\{-\left[(x - \mu)/\sigma\right]^2/2\right\} \; ; \; x > c_p$$

where $p = \Phi\,(c_p - \mu)/\sigma$ is preassigned and represented by the unshaded portion of the normal distribution in figure 1. For a given μ and σ, c_p can be obtained from standard normal tables. From the density function, the expres-

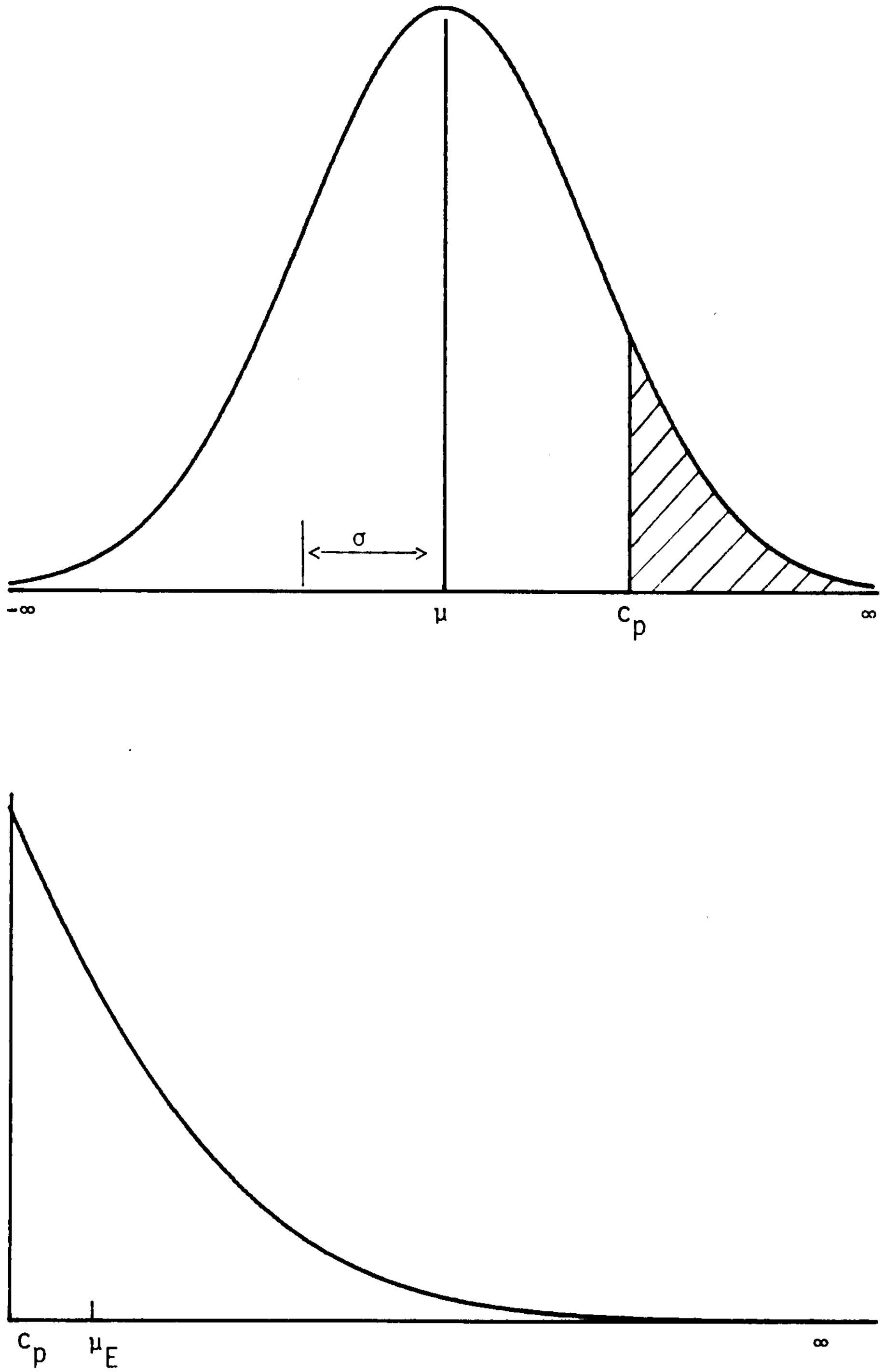

Figure 1. Normal distribution with mean μ and standard deviation σ, and truncated normal distribution truncated at c_p corresponding to p-th percentile.

sions for μ_E and σ_E^2 are

$$\mu_E = [(1 - p)\sigma \sqrt{2\pi}]^{-1} \int_{c_p}^{\infty} x \exp\left\{-[(x - \mu)/\sigma]^2/2\right\} dx$$
$$= \mu + a\sigma$$

$$\sigma_E^2 = [(1 - p)\sigma \sqrt{2\pi}]^{-1} \int_{c_p}^{\infty} x^2 \exp\left\{-[(x - \mu)/\sigma]^2/2\right\} dx - \mu_E^2$$
$$= b\sigma^2$$

where

$$a = [(1 - p)\sqrt{2\pi}]^{-1} \exp\left\{-[(c_p - \mu)/\sigma]^2/2\right\}$$

$$b = [(1 - p)\sqrt{2\pi}]^{-1} \left(\sqrt{2}\left\{\Gamma(3/2) - \int_0^{[(c_p - \mu)/\sigma]^2/2} [\exp(-t)] t^{1/2} dt\right\} - a\right)$$

For any specific value of p, the values of a and b can be obtained with the available tables of complete and incomplete Gamma functions. For example, for $p = .90$, $c_p = 1.282$, the corresponding value of $a = 1.7541$ and $b = (.395)^2$.

Unbiased Estimate of μ_E

It is to be noted that

$$\bar{x} = (\Sigma x_i)/n$$

and

$$s = [\Sigma(x_i - x)^2/(n - 1)]^{\frac{1}{2}}$$

are sufficient statistics from the sample, and must be employed in the estimation of μ_E. Further

$$E(\bar{x}) = \mu \quad ; \quad E(s) = \sigma\sqrt{2/(n - 1)}\, \Gamma(n/2) \Big/ \Gamma\,(n - 1)/2$$

and

$$\mathrm{Var}(\bar{x}) = \sigma^2/n \quad ; \quad \mathrm{Var}(s) = \sigma^2 \left(1 - \left\{\sqrt{2}\Gamma(n/2) / \sqrt{(n - 1)}\, \Gamma[(n - 1)/2]\right\}^2\right)$$

Thus an unbiased estimate of μ_E is given by

$$\hat{\mu}_E = \bar{x} + as\sqrt{(n-1)/2}\,\Gamma\left[(n-1)/2\right]\Big/\Gamma(n/2)$$

$$= \bar{x} + asf$$

where a was derived in the section on truncated distributions, and

$$f = \sqrt{(n-1)/2}\,\Gamma\left[(n-1)/2\right]\Big/\Gamma(n/2)$$

depends on the sample size n via Gamma function values.

Since $\bar{x}$ and s^2 are stochastically independent, the variance of the unbiased estimate μ_E is

$$\begin{aligned}
\mathrm{Var}(\hat{\mu}_E) &= \mathrm{Var}(\bar{x}) + a^2 f^2 \mathrm{Var}(s) \\
&= \sigma^2/n + a^2 f^2 \mathrm{Var}(s) \\
&= \sigma^2/n + a^2 f^2 \sigma^2 (1 - f^{-2}) \\
&= (\sigma^2/n)\left[1 - na^2(f^2 - 1)\right] \\
&= \hat{\sigma}_{\mu_E}^{\;2}
\end{aligned}$$

In this expression, sample size n, a and f are known; and the only unknown factor is σ^2. Thus an estimate of $\mathrm{Var}(\hat{\mu}_E)$ can be obtained by replacing σ^2 by s^2 obtained from the sample. Therefore,

$$\begin{aligned}
\mathrm{Var}(\hat{\mu}_E) &= (s^2/n)\left[1 + na^2(f^2 - 1)\right] \\
&= (s^2/n)(1 + d) \\
&= \hat{\sigma}_{\mu_E}^{\;2}
\end{aligned}$$

This estimate is different from s^2/n by a factor of $d = na^2(f^2 - 1)$, which is a function of n; and even for sufficiently large n, the factor d is not negligible.

Cramer-Rao Lower Bound of Variance of $\hat{\mu}_E$

Variance of the estimate $\hat{\mu}_E$ measures, in some sense, the quality of the unbiased estimate. The smaller the variance, the better the estimate. An unbiased estimate is considered best if it achieves the minimum possible variance without specifying the estimate. The Cramer-Rao lower bound for $\bar{x}$ and s are

$$\mathrm{Var}(\bar{x}) \geq 1/E \quad \frac{\partial}{\partial\mu} \ln f(\bar{x},s)^{\,2} \quad = 1/I_{11}$$

$$\mathrm{Var}(s) \geq 1/E \quad \frac{\partial}{\partial\sigma} \ln f(\bar{x},s)^{\,2} \quad = 1/I_{22}$$

In many cases, it is possible to find an unbiased estimate which achieves this bound. In other cases, it is never attainable. In this section, the lowest bound is obtained and then compared with the variance $\hat{\mu}_E$ computed in the earlier section.

The joint distribution of $\bar{x}$ and s is (ref. 3)

$$f(\bar{x},s) = k_n(s^{(n-2)}/\sigma^n) \exp\left\{ -[(\bar{x} - \mu)\sqrt{n}/\sigma]^2 - (n-1)s^2/\sigma^2\right\}\Big/2$$

where

$$kn = \left[2\sqrt{n}\,(n-1)^{(n-1)/2}\right] \Big/ \left\{\sqrt{2\pi}\,2^{(n-1)/2}\,\Gamma\,[(n-1)/2]\right\}$$

Thus

$$\ln f(\bar{x},s) = \ln k_n + (n-2)\ln s - n \ln\sigma - \left\{[(\bar{x} - \mu)\sqrt{n}/\sigma]^2 + (n-1)s^2/\sigma^2\right\}\Big/2$$

where k_n is the first factor in $f(\bar{x},s)$. From the expression for $\ln f(\bar{x},s)$, it is seen by differentiating that

$$\frac{\partial}{\partial\mu} \ln f(\bar{x},s) = [(\bar{x} - \mu)\sqrt{n}/\sigma]\,(\sqrt{n}/\sigma)$$

$$\frac{\partial}{\partial\sigma} \ln f(\bar{x},s) = -(n/\sigma) + [(\bar{x} - \mu)\sqrt{n}]^2 + \left[(n-1)s^2\right]\left[2/\sigma^3\right]$$

$$= (1/\sigma)\,\left[(\bar{x} - \mu)\sqrt{n}/\sigma\right]^2 + (n-1)s^2/\sigma^2 - n$$

The Cramer-Rao lower bound for $\bar{x}$ and s from these expressions is already given.

$$\mathrm{Var}(\bar{x}) \geq 1/E \left\{(n/\sigma^2)\ [(\bar{x} - \mu)\sqrt{n}/\sigma]^2\right\} = 1/I_{11}$$

$$\mathrm{Var}(s) \geq 1/E \left\{(1/\sigma^2)\ [(\bar{x} - \mu)\sqrt{n}/\sigma]^2\right\} + (n - 1)\ s^2/\sigma^2 - n = 1/I_{22}$$

It is known that the distribution of $[(\bar{x} - \mu)\sqrt{n}/\sigma]^2$ is Chi Square with 1 degree of freedom. The distribution of $(n - 1)\ s^2/\sigma^2$ is Chi Square with $(n - 1)$ degrees of freedom. The two distributions are independent. If Y_r is a Chi Square variable with r degrees of freedom, then $E(Y_r) = r$ and $E(Y_r{}^2) = 2r + r^2$. Therefore

$$I_{11} = (n/\sigma^2)\ E\ [(\bar{x} - \mu)\sqrt{n}/\sigma]^2$$

$$= n/\sigma^2$$

$$I_{22} = (1/\sigma^2)\ E \quad Y_1 + Y_{(n - 1)} \quad - n^2$$

$$= (1/\sigma^2)\ E \quad Y_1{}^2 + Y_{(n - 1)}{}^2 + n^2 + 2Y_1Y_{(n - 1)} - 2n\ Y_1 + Y_{(n - 1)}$$

$$= (1/\sigma^2) \left\{(2 + 1) + \left[2(n - 1) + (n - 1)^2\right] + n^2 + 2(n - 1) - 2n^2\right\}$$

$$= 2n/\sigma^2$$

Thus the lowest bound of the variance of $\hat{\mu}_E = \bar{x} + afs$ is obtained by the appropriate function of inverses of I_{11} and I_{22}. Therefore

$$\mathrm{Var}\ \hat{\mu}_E \geq 1/I_{11} + a^2f^2/I_{22}$$

$$\geq \sigma^2/n + a^2f^2\sigma^2/2n$$

The expression for the variance of $\hat{\mu}_E$ obtained in the earlier section is

$$\mathrm{Var}\ \hat{\mu}_E = \sigma^2/n + a^2\sigma^2(f^2 - 1)$$

which is larger than the lower bound shown above by a factor of $a^2\sigma^2\left[(f^2 - 1) - f^2/2n\right]$. For larger values of n, this factor's value decreases;

therefore, $\hat{\mu}_E$ is a satisfactory estimate of μ_E for all applications. In fact, as n approaches infinity, the variance of the estimate achieves the Cramer-Rao lower bound.

Large Sample Distribution of $T = (\hat{\mu}_E - \mu_E)/\hat{\sigma}_{\mu_E}$

The exact distribution of $\hat{\mu}_E = \bar{x} + afs$ depends on the sum of both the normal and Chi Square distributions. However, for a large sample (n > 30), an approximate distribution is available which can be used to compute confidence bounds.

It is to be noted that the distribution of $(\bar{x} - \mu)\sqrt{n}/\sigma = Z$ is standard normal, that the distribution of $(n - 1)s^2/\sigma^2$ is Chi Square with (n - 1) degrees of freedom, and these distributions are independent of one another. For a large sample ($n \geq 30$), the distribution can be approximated by

$$Z = \sqrt{2Y_{(n-1)}} - \sqrt{2(n-1) - 1}$$

$$= (s/\sigma)\sqrt{2(n-1)} - \sqrt{2n-3}$$

which yields

$$s/\sigma = \left[\sqrt{2(n-1)}\right]^{-1}\left(Z + \sqrt{2n-3}\right)$$

In this section, instead of finding the distribution of $\hat{\mu}_E$, the distribution of the standardized $T = (\hat{\mu}_E - \mu_E)/\hat{\sigma}_{\mu_E}$ is obtained by the above approximation of s/σ.

$$T = (\hat{\mu}_E - \mu_E)/\hat{\sigma}_{\mu_E}$$

$$= \left[(\bar{x} + afs) - (\mu + a\sigma)\right]\Big/ s\sqrt{(1 + d)/n}$$

$$= \left\{(1/\sqrt{n})\left[(x - \mu)\sqrt{n}/\sigma\right]\sigma + a\sigma(fs/\sigma - 1)\right\}\left[\sigma(s/\sigma)\sqrt{(1 + d)/n}\right]^{-1}$$

$$\simeq \frac{(Z/\sqrt{n}) + a\sigma\left\{f\left[(Z + \sqrt{2n-3})\quad\sqrt{2(n-1)}\right] - 1\right\}}{\sqrt{(1 + d)/n}\left[(Z + \sqrt{2n-3})\Big/\sqrt{2(n-1)}\right]}$$

$$\simeq \frac{Z\left[(1/\sqrt{n}) + af/\sqrt{2(n-1)}\right] + a\left[f\sqrt{(2n-3)/(2n-2)} - 1\right]}{\sqrt{(1 + d)/2n(n-1)}\left(Z + \sqrt{2n-3}\right)}$$

$$\simeq (ZA + B)\big/C(Z + D)$$

where

$$A = (Z/\sqrt{n}) + af/\sqrt{2(n - 1)}$$

$$B = a(f\sqrt{(2n - 3)/(2n - 2)} - 1)$$

$$C = \sqrt{(1 + d)/2n(n - 1)}$$

$$D = \sqrt{2n - 3}$$

are functions of the sample size n and a preassigned value of p. Since Z has a standard normal density, the density function of T is given by

$$D(t) = \left[C(AD - B)/\sqrt{2\pi}\right](A - Ct)^{-2} \exp\left\{-[(CDt - B)/(A - Ct)]^2/2\right\}$$

This density function does not depend on μ or σ, but is a function of the sample size n and the preassigned value of p. Appendix B lists the computer program which generates the density and distribution function of T for a designated sample size n and value of p. A sample of the density of T for n = 90 and p = .90 is tabulated in Table 1 and shown in figure 2. Figure 2 also contrasts the density of T with a standard normal density.

Confidence Intervals of μ_E

A confidence interval is either specified by assigning a level of confidence α, or by assigning the length of the confidence interval 2m. In the first case, lower and upper confidence bounds L and U are obtained which, in a long series of experiments, are likely to include the population μ_E, α% of times. In the second case, the values of L and U are fixed and the level α is obtained. Analytically both the cases involve solving for either L, U or α, given the other.

<u>Confidence interval with α confidence</u>.- In this case, the equation

$$\Pr\left[1 < (ZA + B)/C(Z + D) < u\right] = \alpha$$

needs to be solved. Since Z is a standard normal variate, it follows that

$$1 = \left(-Z_{\alpha/2}A + B\right)\Big/\left[C(-Z_{\alpha/2}) + D\right]$$

$$u = \left(Z_{\alpha/2}A + B\right)\Big/\left(CZ_{\alpha/2} + D\right)$$

where $Z_{\alpha/2}$ is obtained from a normal probability table. For these expressions, it is seen that the lower and upper confidence values, L and U, for μ_E are

$$L = \bar{x} + afs - us\sqrt{(1 + d)/n}$$

$$U = \bar{x} + afs + 1s\sqrt{(1 + d)/n}$$

TABLE I. DENSITY (D(T)) AND DISTRIBUTION (F(T)) OF THE RANDOM VARIABLE
T = (MU HAT - MU)/S(MU HAT)   N = 90

T	D(T)	F(T)	T	D(T)	F(T)
-5.00	.003591	.002637	0.00	.281669	.500024
-4.90	.004093	.003020	.10	.283965	.528317
-4.80	.004662	.003457	.20	.284827	.556769
-4.70	.005304	.003955	.30	.284193	.585232
-4.60	.006029	.004521	.40	.282027	.613556
-4.50	.006846	.005164	.50	.278312	.641586
-4.40	.007766	.005894	.60	.273062	.669168
-4.30	.008799	.006721	.70	.266315	.696149
-4.20	.009958	.007658	.80	.258136	.722383
-4.10	.011257	.008717	.90	.248620	.747731
-4.00	.012709	.009914	1.00	.237884	.772066
-3.90	.014329	.011265	1.10	.226069	.795272
-3.80	.016135	.012746	1.20	.213339	.817249
-3.70	.018143	.014498	1.30	.199872	.837915
-3.60	.020371	.016422	1.40	.185859	.857206
-3.50	.022840	.018581	1.50	.171496	.875075
-3.40	.025557	.020999	1.60	.156985	.891500
-3.30	.028575	.023703	1.70	.142521	.906474
-3.20	.031884	.026724	1.80	.128292	.920012
-3.10	.035515	.030091	1.90	.114471	.932146
-3.00	.039489	.033838	2.00	.101214	.942925
-2.90	.043827	.038001	2.10	.088656	.952412
-2.80	.048549	.042616	2.20	.076905	.960683
-2.70	.053673	.047724	2.30	.066046	.967823
-2.60	.059218	.053365	2.40	.056136	.973924
-2.50	.065199	.059582	2.50	.047204	.979083
-2.40	.071626	.066420	2.60	.039256	.983398
-2.30	.078510	.073923	2.70	.032275	.986966
-2.20	.085856	.082137	2.80	.026223	.989883
-2.10	.093662	.091109	2.90	.021047	.992240
-2.00	.101924	.100885	3.00	.016680	.994120
-1.90	.110628	.111509	3.10	.013048	.995600
-1.80	.119757	.123024	3.20	.010070	.996751
-1.70	.129283	.135473	3.30	.007663	.997633
-1.60	.139170	.148893	3.40	.005748	.998300
-1.50	.149374	.163318	3.50	.004248	.998797
-1.40	.159839	.178777	3.60	.003091	.999161
-1.30	.170502	.195292	3.70	.002213	.999424
-1.20	.181287	.212881	3.80	.001559	.999611
-1.10	.192110	.231551	3.90	.001079	.999742
-1.00	.202877	.251301	4.00	.000734	.999832
-.90	.213482	.272121	4.10	.000490	.999892
-.80	.223815	.293989	4.20	.000321	.999932
-.70	.233755	.316871	4.30	.000206	.999958
-.60	.243178	.340722	4.40	.000130	.999975
-.50	.251954	.365485	4.50	.000080	.999985
-.40	.259953	.391087	4.60	.000048	.999991
-.30	.267046	.417445	4.70	.000028	.999995
-.20	.273107	.444462	4.80	.000016	.999997
-.10	.278018	.472028	4.90	.000009	.999999

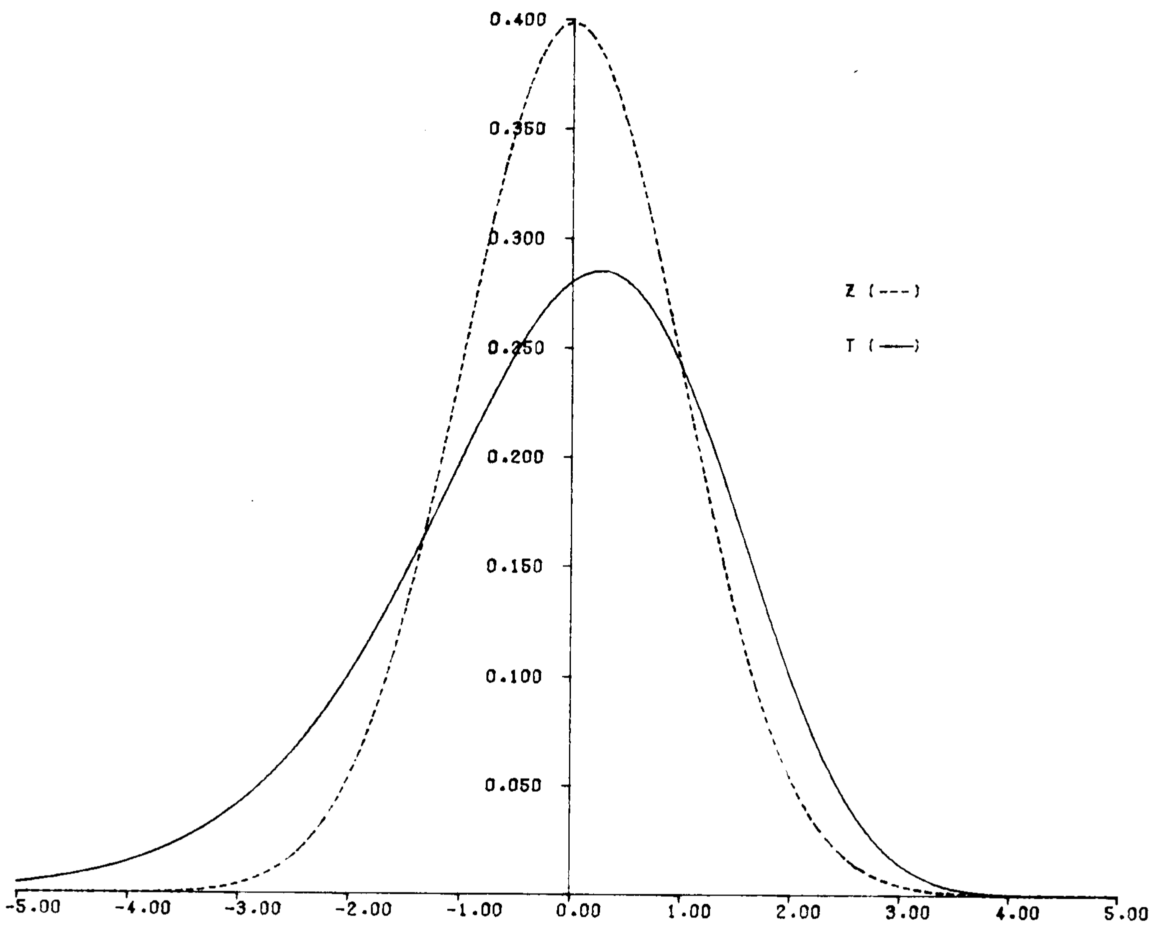

Figure 2. Density function of the standard normal variate Z, and the density function of the standardized extreme mean variate T.

Confidence interval of fixed length 2m.- In this case, it is already known that $L = \mu_E - m$ and $U = \mu_E + m$. The object, therefore, is to find the corresponding value of α_m. Since $\sigma_{\mu_E} = s\sqrt{(1 + d)/n}$ in the terms of standardized T variable, it is required to find

$$\alpha_m = \Pr\left[-m/s\sqrt{(1 + d)/n} < T < m/s\sqrt{(1 + d)/n}\right]$$

$$\alpha_m = \Pr\left[-m\big/s\sqrt{(1 + d)/n} < (ZA + B)/C(Z + D) < m\big/s\sqrt{(1 + d)/n}\right]$$

$$= \Pr\left[Z_L < Z < Z_U\right]$$

$$= \Phi(Z_U) - \Phi(Z_L)$$

The right-hand side can be read from standard normal probability tables after calculating Z_L and Z_U which are given below.

$$Z_L = mCD/s\sqrt{(1 + d)/n} + B \Big/ A + mC/s\sqrt{(1 + d)/n}$$

$$Z_U = mCD/s\sqrt{(1 + d)/n} - B \Big/ A - mC/s\sqrt{(1 + d)/n}$$

A table of confidence bounds for standardized T for various sample sizes are given in Appendix A for ready use.

APPLICATION TO FLIGHT DATA

To demonstrate the usefulness of extreme mean estimation, data are obtained on distortion parameter IDT, K_A, K_Θ, and K_r from high frequency response pressure measurements made at the inlet/engine interface plane during a supersonic aircraft propulsion research program (ref. 1). The summarized data are presented in Table 2. The data show the sample size, the sample mean ($\bar{x}$) and standard deviation ($\hat{\sigma}$) for all distortion parameters.

The succeeding four tables, numbered 3, 4, 5, and 6, show the extreme means and their lower and upper 95% confidence bounds for $\alpha = .95$. Tables 7, 8, 9, and 10 present corresponding values for $\alpha = .99$.

TABLE 2. SUMMARY OF DISTORTION PARAMETERS' DATA

SAMPLE #	SAMPLE SIZE	IDT $\bar{x}$	IDT $\hat{\sigma}$	K_A $\bar{x}$	K_A $\hat{\sigma}$	K_θ $\bar{x}$	K_θ $\hat{\sigma}$	K_r $\bar{x}$	K_r $\hat{\sigma}$
1	470	.137	.012	.875	.084	.377	.068	.498	.049
2	470	.167	.013	1.285	.119	.767	.101	.518	.067
3	470	.194	.017	1.050	.082	.400	.065	.650	.050
4	470	.207	.016	1.088	.081	.360	.074	.729	.056
5	470	.225	.018	1.170	.095	.443	.076	.727	.062
6	406	.118	.012	.744	.071	.414	.060	.330	.038
7	406	.120	.014	.572	.052	.297	.043	.275	.029
8	406	.186	.032	.886	.149	.519	.128	.367	.106
9	406	.177	.015	.834	.079	.264	.059	.570	.043
10	406	.200	.019	.891	.109	.373	.074	.518	.057
11	448	.194	.012	1.075	.093	.377	.078	.697	.055
12	448	.226	.013	1.153	.103	.415	.090	.738	.057
13	448	.257	.027	1.514	.136	.541	.112	.973	.093
14	448	.184	.014	.682	.058	.302	.047	.381	.036
15	448	.206	.015	.731	.074	.314	.059	.418	.043

TABLE 3. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.06 SIGMA (DATA: IDT)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.1370	.0120	.1596	.1641
2	470	.1670	.0130	.1915	.1964
3	470	.1940	.0170	.2261	.2324
4	470	.2070	.0160	.2372	.2432
5	470	.2250	.0180	.2590	.2657
6	406	.1160	.0120	.1405	.1453
7	406	.1200	.0140	.1462	.1519
8	406	.1860	.0320	.2460	.2589
9	406	.1770	.0150	.2051	.2112
10	406	.2000	.0190	.2356	.2433
11	448	.1940	.0120	.2166	.2212
12	448	.2260	.0130	.2505	.2555
13	448	.2570	.0270	.3073	.3182
14	448	.1840	.0140	.2104	.2157
15	448	.2060	.0150	.2342	.2400

TABLE 4. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.06 SIGMA (DATA: K_A)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.8750	.0840	1.0335	1.0649
2	470	1.2850	.1190	1.5095	1.5541
3	470	1.0500	.0820	1.2047	1.2354
4	470	1.0880	.0810	1.2408	1.2712
5	470	1.1700	.0950	1.3492	1.3848
6	406	.7440	.0710	.8770	.9057
7	406	.5720	.0520	.6694	.6904
8	406	.8860	.1490	1.1652	1.2253
9	406	.8340	.0790	.9820	1.0139
10	406	.8910	.1090	1.0953	1.1392
11	448	1.0750	.0930	1.2501	1.2858
12	448	1.1530	.1030	1.3469	1.3864
13	448	1.5140	.1360	1.7700	1.8222
14	448	.6820	.0580	.7912	.8134
15	448	.7310	.0740	.8703	.8987

TABLE 5. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.06 SIGMA (DATA: K_θ)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.3770	.0680	.5053	.5308
2	470	.7670	.1010	.9575	.9954
3	470	.4000	.0650	.5226	.5470
4	470	.3600	.0750	.5015	.5296
5	470	.4430	.0760	.5864	.6149
6	406	.4140	.0600	.5264	.5506
7	406	.2970	.0430	.3776	.3949
8	406	.5190	.1280	.7589	.8105
9	406	.2640	.0590	.3746	.3984
10	406	.3730	.0740	.5117	.5415
11	448	.3770	.0780	.5238	.5538
12	448	.4150	.0900	.5844	.6190
13	448	.5410	.1120	.7518	.7948
14	448	.3020	.0470	.3905	.4085
15	448	.3140	.0590	.4251	.4477

TABLE 6. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.06 SIGMA (DATA: K_R)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.4980	.0490	.5904	.6088
2	470	.5180	.0670	.6444	.6695
3	470	.6500	.0500	.7443	.7631
4	470	.7290	.0560	.8346	.8556
5	470	.7270	.0620	.8440	.8672
6	406	.3300	.0380	.4012	.4165
7	406	.2750	.0290	.3293	.3410
8	406	.3670	.1060	.5656	.6084
9	406	.5700	.0430	.6506	.6679
10	406	.5180	.0570	.6248	.6478
11	448	.6970	.0550	.8005	.8217
12	448	.7380	.0570	.8453	.8672
13	448	.9730	.0930	1.1481	1.1838
14	448	.3810	.0360	.4488	.4626
15	448	.4180	.0430	.4989	.5155

TABLE 7. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.67 SIGMA (DATA: IDT)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.1370	.0120	.1655	.1730
2	470	.1670	.0130	.1979	.2060
3	470	.1940	.0170	.2344	.2450
4	470	.2070	.0160	.2451	.2550
5	470	.2250	.0180	.2673	.2790
6	406	.1180	.0120	.1463	.1544
7	406	.1200	.0140	.1530	.1624
8	406	.1660	.0320	.2615	.2830
9	406	.1770	.0150	.2124	.2225
10	406	.2000	.0190	.2448	.2576
11	448	.1940	.0120	.2225	.2301
12	448	.2260	.0130	.2568	.2651
13	448	.2570	.0270	.3210	.3383
14	448	.1840	.0140	.2172	.2262
15	448	.2060	.0150	.2416	.2512

TABLE 8. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.67 SIGMA (DATA: K_A)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.8750	.0840	1.0748	1.1272
2	470	1.2850	.1190	1.5680	1.6423
3	470	1.0500	.0820	1.2450	1.2962
4	470	1.0880	.0810	1.2807	1.3312
5	470	1.1700	.0950	1.3960	1.4552
6	406	.7440	.0710	.9114	.9591
7	406	.5720	.0520	.6946	.7296
8	406	.8860	.1490	1.2374	1.3375
9	406	.8340	.0790	1.0203	1.0734
10	406	.8910	.1090	1.1481	1.2213
11	448	1.0750	.0930	1.2956	1.3550
12	448	1.1530	.1030	1.3973	1.4631
13	448	1.5140	.1360	1.8366	1.9235
14	448	.6820	.0580	.8196	.8566
15	448	.7310	.0740	.9065	.9538

TABLE 9. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.67 SIGMA (DATA: K_θ)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.3770	.0680	.5387	.5811
2	470	.7670	.1010	1.0072	1.0702
3	470	.4000	.0650	.5546	.5951
4	470	.3600	.0750	.5384	.5852
5	470	.4430	.0760	.6238	.6712
6	406	.4140	.0600	.5555	.5852
7	406	.2970	.0430	.3984	.4273
8	406	.5190	.1280	.8209	.9069
9	406	.2640	.0590	.4031	.4428
10	406	.3730	.0740	.5475	.5972
11	448	.3770	.0780	.5620	.6119
12	448	.4150	.0900	.6285	.6860
13	448	.5410	.1120	.8067	.8782
14	448	.3020	.0470	.4135	.4435
15	448	.3140	.0590	.4540	.4917

TABLE 10. 95 PERCENT CONFIDENCE INTERVALS FOR EXTREME MEAN, MU + 2.67 SIGMA (DATA: K_R)

SAMPLE NUMBER	SAMPLE SIZE	SAMPLE MEAN	SAMPLE STD DEV	LOWER BOUND	UPPER BOUND
1	470	.4980	.0490	.6145	.6451
2	470	.5180	.0670	.6774	.7191
3	470	.6500	.0500	.7689	.8001
4	470	.7290	.0560	.8622	.8971
5	470	.7270	.0620	.8745	.9131
6	406	.3300	.0380	.4196	.4451
7	406	.2750	.0290	.3434	.3629
8	406	.3670	.1060	.6170	.6882
9	406	.5700	.0430	.6714	.7003
10	406	.5180	.0570	.6524	.6907
11	448	.6970	.0550	.8275	.8626
12	448	.7380	.0570	.8732	.9096
13	448	.9730	.0930	1.1936	1.2530
14	448	.3810	.0360	.4664	.4894
15	448	.4180	.0430	.5200	.5475

APPENDIX A

This appendix presents upper and lower confidence bounds of T for α = .90, .95, .975, .99 and various sample sizes.

TABLE 11. UPPER AND LOWER CONFIDENCE BOUNDS OF T =(MU HAT - MU)/S(MU)
FOR ALPHA =0.90, 0.95, 0.975, AND 0.99

p=.9

UPPER BOUND = (B+A*Z)/(C*(D+Z)), LOWER BOUND = (B-A*Z)/(C*(D-Z))
(Z VALUES ARE STANDARD NORMAL VALUES FOR ALPHA)

SAMPLE	ALPHA = 0.90		ALPHA = 0.95		ALPHA = 0.975		ALPHA = 0.99	
SIZE	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND
30	1.9409	-3.0236	2.2361	-3.8055	2.4825	-4.5783	2.7602	-5.6207
40	1.9798	-2.8941	2.2898	-3.6076	2.5504	-4.2996	2.8462	-5.2125
50	2.0079	-2.8137	2.3287	-3.4863	2.5997	-4.1309	2.9088	-4.9700
60	2.0296	-2.7580	2.3586	-3.4030	2.6378	-4.0160	2.9574	-4.8067
61	2.0315	-2.7533	2.3613	-3.3960	2.6411	-4.0065	2.9617	-4.7932
62	2.0333	-2.7488	2.3639	-3.3892	2.6444	-3.9972	2.9658	-4.7801
63	2.0352	-2.7444	2.3664	-3.3827	2.6476	-3.9882	2.9700	-4.7674
64	2.0370	-2.7401	2.3689	-3.3763	2.6508	-3.9795	2.9740	-4.7551
65	2.0387	-2.7359	2.3713	-3.3701	2.6539	-3.9710	2.9779	-4.7431
66	2.0404	-2.7319	2.3737	-3.3641	2.6569	-3.9627	2.9818	-4.7314
67	2.0421	-2.7279	2.3760	-3.3582	2.6599	-3.9547	2.9856	-4.7201
68	2.0437	-2.7241	2.3783	-3.3525	2.6628	-3.9468	2.9893	-4.7091
69	2.0454	-2.7203	2.3805	-3.3469	2.6656	-3.9392	2.9929	-4.6984
70	2.0469	-2.7167	2.3827	-3.3415	2.6684	-3.9318	2.9965	-4.6880
71	2.0485	-2.7131	2.3849	-3.3362	2.6712	-3.9246	3.0000	-4.6778
72	2.0500	-2.7096	2.3870	-3.3311	2.6739	-3.9175	3.0035	-4.6679
73	2.0515	-2.7062	2.3891	-3.3260	2.6765	-3.9107	3.0069	-4.6583
74	2.0530	-2.7029	2.3911	-3.3211	2.6791	-3.9040	3.0102	-4.6489
75	2.0544	-2.6997	2.3931	-3.3163	2.6817	-3.8974	3.0135	-4.6397
76	2.0559	-2.6965	2.3951	-3.3116	2.6842	-3.8910	3.0167	-4.6308
77	2.0573	-2.6934	2.3970	-3.3171	2.6867	-3.8848	3.0198	-4.6220
78	2.0586	-2.6904	2.3989	-3.3026	2.6891	-3.8787	3.0230	-4.6135
79	2.0600	-2.6875	2.4008	-3.2982	2.6915	-3.8727	3.0260	-4.6052
80	2.0613	-2.6846	2.4026	-3.2939	2.6938	-3.8669	3.0290	-4.5970
81	2.0626	-2.6817	2.4045	-3.2898	2.6961	-3.8612	3.0320	-4.5891
82	2.0639	-2.6790	2.4062	-3.2857	2.6984	-3.8556	3.0349	-4.5813
83	2.0652	-2.6763	2.4080	-3.2816	2.7007	-3.8502	3.0378	-4.5737
84	2.0664	-2.6736	2.4097	-3.2777	2.7029	-3.8449	3.0406	-4.5662
85	2.0676	-2.6710	2.4114	-3.2739	2.7050	-3.8396	3.0433	-4.5590
86	2.0688	-2.6684	2.4131	-3.2701	2.7072	-3.8345	3.0461	-4.5518
87	2.0700	-2.6659	2.4147	-3.2664	2.7093	-3.8295	3.0488	-4.5448
88	2.0712	-2.6635	2.4164	-3.2628	2.7113	-3.8246	3.0514	-4.5380
89	2.0723	-2.6611	2.4180	-3.2593	2.7134	-3.8198	3.0540	-4.5313
90	2.0734	-2.6587	2.4195	-3.2558	2.7154	-3.8151	3.0566	-4.5247
91	2.0746	-2.6564	2.4211	-3.2524	2.7173	-3.8104	3.0592	-4.5183
92	2.0757	-2.6541	2.4226	-3.2490	2.7193	-3.8059	3.0617	-4.5120
93	2.0767	-2.6519	2.4241	-3.2458	2.7212	-3.8014	3.0641	-4.5058
94	2.0778	-2.6497	2.4256	-3.2425	2.7231	-3.7971	3.0665	-4.4997
95	2.0789	-2.6476	2.4271	-3.2394	2.7250	-3.7928	3.0689	-4.4937
100	2.0839	-2.6374	2.4341	-3.2244	2.7339	-3.7725	3.0805	-4.4655
120	2.1011	-2.6040	2.4580	-3.1754	2.7646	-3.7062	3.1198	-4.3738

TABLE 11.-CONTINUED

SAMPLE SIZE	ALPHA = 0.90 UPPER BOUND	ALPHA = 0.90 LOWER BOUND	ALPHA = 0.95 UPPER BOUND	ALPHA = 0.95 LOWER BOUND	ALPHA = 0.975 UPPER BOUND	ALPHA = 0.975 LOWER BOUND	ALPHA = 0.99 UPPER BOUND	ALPHA = 0.99 LOWER BOUND
140	2.1178	-2.5827	2.4806	-3.1432	2.7930	-3.6620	3.1558	-4.3119
160	2.1287	-2.5623	2.4959	-3.1137	2.8126	-3.6225	3.1812	-4.2577
180	2.1379	-2.5458	2.5086	-3.0897	2.8292	-3.5905	3.2027	-4.2141
200	2.1457	-2.5321	2.5199	-3.0699	2.8435	-3.5640	3.2212	-4.1779
220	2.1526	-2.5205	2.5295	-3.0531	2.8559	-3.5416	3.2373	-4.1474
240	2.1587	-2.5105	2.5380	-3.0386	2.8669	-3.5223	3.2515	-4.1213
260	2.1641	-2.5018	2.5456	-3.0260	2.8767	-3.5055	3.2642	-4.0985
280	2.1689	-2.4941	2.5524	-3.0149	2.8855	-3.4907	3.2757	-4.0785
300	2.1733	-2.4872	2.5586	-3.0050	2.8934	-3.4776	3.2860	-4.0607
320	2.1773	-2.4810	2.5642	-2.9961	2.9007	-3.4658	3.2955	-4.0448
340	2.1810	-2.4755	2.5694	-2.9881	2.9073	-3.4552	3.3042	-4.0304
360	2.1844	-2.4704	2.5741	-2.9808	2.9135	-3.4455	3.3121	-4.0173
380	2.1875	-2.4658	2.5785	-2.9741	2.9191	-3.4367	3.3195	-4.0054
400	2.1904	-2.4615	2.5826	-2.9680	2.9244	-3.4286	3.3264	-3.9945
420	2.1931	-2.4576	2.5864	-2.9624	2.9293	-3.4211	3.3328	-3.9844
440	2.1956	-2.4539	2.5899	-2.9571	2.9339	-3.4141	3.3388	-3.9751
460	2.1980	-2.4505	2.5933	-2.9523	2.9382	-3.4077	3.3444	-3.9664
480	2.2002	-2.4474	2.5964	-2.9477	2.9423	-3.4017	3.3497	-3.9583
500	2.2023	-2.4444	2.5994	-2.9435	2.9461	-3.3961	3.3547	-3.9507
520	2.2043	-2.4417	2.6022	-2.9395	2.9497	-3.3908	3.3594	-3.9437
540	2.2062	-2.4390	2.6048	-2.9358	2.9532	-3.3859	3.3639	-3.9370
560	2.2079	-2.4366	2.6073	-2.9322	2.9564	-3.3812	3.3681	-3.9307
580	2.2096	-2.4343	2.6097	-2.9289	2.9595	-3.3768	3.3722	-3.9248
600	2.2112	-2.4321	2.6120	-2.9257	2.9624	-3.3726	3.3760	-3.9192
620	2.2128	-2.4300	2.6141	-2.9227	2.9652	-3.3687	3.3797	-3.9139
640	2.2142	-2.4280	2.6162	-2.9199	2.9679	-3.3649	3.3832	-3.9089
660	2.2157	-2.4261	2.6182	-2.9172	2.9705	-3.3613	3.3865	-3.9041
680	2.2170	-2.4243	2.6201	-2.9146	2.9729	-3.3579	3.3897	-3.8995
700	2.2183	-2.4226	2.6219	-2.9121	2.9753	-3.3547	3.3928	-3.8951
720	2.2195	-2.4209	2.6236	-2.9098	2.9775	-3.3516	3.3958	-3.8909
740	2.2207	-2.4193	2.6253	-2.9075	2.9797	-3.3486	3.3986	-3.8870
760	2.2219	-2.4178	2.6269	-2.9054	2.9818	-3.3457	3.4013	-3.8831
780	2.2230	-2.4164	2.6285	-2.9033	2.9838	-3.3430	3.4040	-3.8795
800	2.2240	-2.4150	2.6300	-2.9013	2.9858	-3.3404	3.4065	-3.8760
820	2.2250	-2.4137	2.6314	-2.8994	2.9876	-3.3379	3.4090	-3.8726
840	2.2260	-2.4124	2.6328	-2.8975	2.9894	-3.3354	3.4113	-3.8693
860	2.2270	-2.4111	2.6342	-2.8958	2.9912	-3.3331	3.4136	-3.8662
880	2.2279	-2.4099	2.6355	-2.8941	2.9929	-3.3308	3.4158	-3.8632
900	2.2288	-2.4088	2.6367	-2.8924	2.9945	-3.3287	3.4180	-3.8603
920	2.2297	-2.4077	2.6380	-2.8908	2.9961	-3.3266	3.4200	-3.8574
940	2.2305	-2.4066	2.6391	-2.8893	2.9976	-3.3245	3.4220	-3.8547

TABLE 12. UPPER AND LOWER CONFIDENCE BOUNDS OF T =(MU HAT - MU)/S(MU)
FOR ALPHA = 0.90, 0.95, 0.975, AND 0.99
P=.95
UPPER BOUND = (B+A*Z)/(C*(D+Z)), LOWER BOUND = (B-A*Z)/(C*(D-Z))
(Z VALUES ARE STANDARD NORMAL VALUES FOR ALPHA)

SAMPLE	ALPHA = 0.90		ALPHA = 0.95		ALPHA = 0.975		ALPHA = 0.99	
SIZE	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND
30	1.9173	-2.9868	2.2088	-3.7591	2.4522	-4.5226	2.7265	-5.5522
40	1.9561	-2.8595	2.2623	-3.5645	2.5198	-4.2481	2.8121	-5.1501
50	1.9842	-2.7804	2.3011	-3.4450	2.5689	-4.0820	2.8744	-4.9112
60	2.0057	-2.7256	2.3309	-3.3630	2.6068	-3.9688	2.9226	-4.7502
61	2.0076	-2.7210	2.3335	-3.3561	2.6101	-3.9594	2.9269	-4.7369
62	2.0095	-2.7165	2.3361	-3.3495	2.6134	-3.9503	2.9310	-4.7240
63	2.0113	-2.7122	2.3386	-3.3430	2.6166	-3.9414	2.9351	-4.7115
64	2.0131	-2.7080	2.3411	-3.3367	2.6197	-3.9328	2.9391	-4.6993
65	2.0148	-2.7039	2.3435	-3.3306	2.6228	-3.9244	2.9430	-4.6875
66	2.0165	-2.6999	2.3458	-3.3247	2.6258	-3.9163	2.9468	-4.6760
67	2.0182	-2.6960	2.3482	-3.3189	2.6287	-3.9084	2.9506	-4.6649
68	2.0198	-2.6922	2.3504	-3.3133	2.6316	-3.9007	2.9543	-4.6540
69	2.0214	-2.6885	2.3527	-3.3078	2.6344	-3.8932	2.9579	-4.6435
70	2.0230	-2.6849	2.3548	-3.3024	2.6372	-3.8859	2.9615	-4.6332
71	2.0246	-2.6814	2.3570	-3.2972	2.6400	-3.8787	2.9650	-4.6232
72	2.0261	-2.6780	2.3591	-3.2922	2.6426	-3.8718	2.9684	-4.6134
73	2.0276	-2.6747	2.3612	-3.2872	2.6453	-3.8650	2.9718	-4.6039
74	2.0290	-2.6714	2.3632	-3.2824	2.6479	-3.8584	2.9751	-4.5947
75	2.0305	-2.6682	2.3652	-3.2776	2.6504	-3.8520	2.9783	-4.5856
76	2.0319	-2.6651	2.3672	-3.2730	2.6529	-3.8457	2.9815	-4.5768
77	2.0333	-2.6620	2.3691	-3.2685	2.6553	-3.8395	2.9846	-4.5682
78	2.0346	-2.6591	2.3710	-3.2641	2.6578	-3.8335	2.9877	-4.5598
79	2.0360	-2.6562	2.3728	-3.2598	2.6601	-3.8276	2.9908	-4.5515
80	2.0373	-2.6533	2.3747	-3.2556	2.6625	-3.8219	2.9937	-4.5435
81	2.0386	-2.6505	2.3765	-3.2515	2.6648	-3.8163	2.9967	-4.5357
82	2.0399	-2.6478	2.3782	-3.2474	2.6670	-3.8108	2.9996	-4.5280
83	2.0411	-2.6451	2.3801	-3.2435	2.6692	-3.8054	3.0024	-4.5205
84	2.0424	-2.6425	2.3817	-3.2396	2.6714	-3.8002	3.0052	-4.5131
85	2.0436	-2.6400	2.3834	-3.2358	2.6736	-3.7950	3.0080	-4.5060
86	2.0448	-2.6374	2.3850	-3.2321	2.6757	-3.7900	3.0107	-4.4989
87	2.0459	-2.6350	2.3867	-3.2285	2.6778	-3.7850	3.0133	-4.4920
88	2.0471	-2.6326	2.3883	-3.2249	2.6798	-3.7802	3.0160	-4.4853
89	2.0482	-2.6302	2.3899	-3.2214	2.6819	-3.7754	3.0186	-4.4787
90	2.0494	-2.6279	2.3914	-3.2180	2.6838	-3.7708	3.0211	-4.4722
91	2.0505	-2.6256	2.3930	-3.2146	2.6858	-3.7662	3.0236	-4.4658
92	2.0516	-2.6234	2.3945	-3.2114	2.6877	-3.7617	3.0261	-4.4596
93	2.0527	-2.6212	2.3960	-3.2081	2.6897	-3.7574	3.0286	-4.4535
94	2.0537	-2.6190	2.3975	-3.2049	2.6915	-3.7530	3.0310	-4.4475
95	2.0548	-2.6169	2.3989	-3.2018	2.6934	-3.7488	3.0334	-4.4416
100	2.0598	-2.6069	2.4059	-3.1871	2.7023	-3.7288	3.0448	-4.4138
120	2.0769	-2.5740	2.4297	-3.1387	2.7327	-3.6635	3.0838	-4.3234

TABLE 12.-CONTINUED

SAMPLE SIZE	ALPHA = 0.90 UPPER BOUND	ALPHA = 0.90 LOWER BOUND	ALPHA = 0.95 UPPER BOUND	ALPHA = 0.95 LOWER BOUND	ALPHA = 0.975 UPPER BOUND	ALPHA = 0.975 LOWER BOUND	ALPHA = 0.99 UPPER BOUND	ALPHA = 0.99 LOWER BOUND
140	2.0938	-2.5534	2.4525	-3.1076	2.7613	-3.6206	3.1200	-4.2631
160	2.1046	-2.5333	2.4676	-3.0784	2.7807	-3.5814	3.1452	-4.2095
180	2.1136	-2.5170	2.4804	-3.0549	2.7972	-3.5498	3.1664	-4.1663
200	2.1214	-2.5035	2.4913	-3.0351	2.8113	-3.5236	3.1847	-4.1306
220	2.1282	-2.4920	2.5009	-3.0185	2.8235	-3.5015	3.2006	-4.1005
240	2.1342	-2.4821	2.5093	-3.0042	2.8344	-3.4824	3.2147	-4.0746
260	2.1396	-2.4734	2.5168	-2.9917	2.8441	-3.4658	3.2273	-4.0521
280	2.1444	-2.4658	2.5235	-2.9807	2.8528	-3.4512	3.2386	-4.0323
300	2.1487	-2.4590	2.5296	-2.9710	2.8607	-3.4382	3.2488	-4.0147
320	2.1527	-2.4529	2.5352	-2.9622	2.8678	-3.4266	3.2582	-3.9990
340	2.1563	-2.4474	2.5403	-2.9543	2.8744	-3.4161	3.2667	-3.9848
360	2.1596	-2.4424	2.5450	-2.9471	2.8805	-3.4065	3.2746	-3.9719
380	2.1627	-2.4379	2.5493	-2.9405	2.8861	-3.3978	3.2820	-3.9601
400	2.1656	-2.4336	2.5534	-2.9344	2.8913	-3.3897	3.2887	-3.9493
420	2.1683	-2.4298	2.5571	-2.9288	2.8962	-3.3823	3.2951	-3.9393
440	2.1708	-2.4262	2.5606	-2.9237	2.9007	-3.3755	3.3010	-3.9301
460	2.1731	-2.4228	2.5639	-2.9189	2.9050	-3.3691	3.3066	-3.9215
480	2.1753	-2.4197	2.5670	-2.9144	2.9090	-3.3632	3.3118	-3.9135
500	2.1774	-2.4168	2.5699	-2.9102	2.9128	-3.3576	3.3167	-3.9060
520	2.1793	-2.4140	2.5727	-2.9062	2.9163	-3.3524	3.3214	-3.8990
540	2.1812	-2.4114	2.5753	-2.9025	2.9197	-3.3475	3.3258	-3.8924
560	2.1829	-2.4090	2.5778	-2.8990	2.9229	-3.3429	3.3300	-3.8862
580	2.1846	-2.4067	2.5802	-2.8957	2.9260	-3.3386	3.3340	-3.8804
600	2.1862	-2.4045	2.5824	-2.8926	2.9289	-3.3344	3.3378	-3.8748
620	2.1877	-2.4025	2.5846	-2.8897	2.9317	-3.3305	3.3414	-3.8696
640	2.1892	-2.4005	2.5866	-2.8868	2.9343	-3.3268	3.3449	-3.8646
660	2.1906	-2.3986	2.5886	-2.8842	2.9369	-3.3233	3.3482	-3.8599
680	2.1919	-2.3968	2.5904	-2.8816	2.9393	-3.3199	3.3514	-3.8553
700	2.1932	-2.3951	2.5922	-2.8792	2.9416	-3.3167	3.3544	-3.8510
720	2.1944	-2.3935	2.5939	-2.8768	2.9438	-3.3136	3.3573	-3.8469
740	2.1956	-2.3920	2.5956	-2.8746	2.9460	-3.3107	3.3601	-3.8430
760	2.1967	-2.3905	2.5972	-2.8725	2.9481	-3.3079	3.3628	-3.8392
780	2.1978	-2.3890	2.5987	-2.8704	2.9500	-3.3052	3.3654	-3.8356
800	2.1988	-2.3877	2.6002	-2.8685	2.9520	-3.3026	3.3679	-3.8321
820	2.1999	-2.3863	2.6016	-2.8666	2.9538	-3.3001	3.3704	-3.8287
840	2.2008	-2.3851	2.6030	-2.8647	2.9556	-3.2977	3.3727	-3.8255
860	2.2018	-2.3838	2.6044	-2.8630	2.9573	-3.2954	3.3750	-3.8224
880	2.2027	-2.3827	2.6056	-2.8613	2.9590	-3.2931	3.3772	-3.8194
900	2.2036	-2.3815	2.6069	-2.8597	2.9606	-3.2910	3.3793	-3.8166
920	2.2044	-2.3804	2.6081	-2.8581	2.9622	-3.2889	3.3813	-3.8138
940	2.2053	-2.3794	2.6093	-2.8566	2.9637	-3.2869	3.3833	-3.8111

TABLE 13. UPPER AND LOWER CONFIDENCE BOUNDS OF T =(MU HAT - MU)/S(MU)
FOR ALPHA =0.90, 0.95, 0.975, AND 0.99
p=.975
UPPER BOUND = (B+A*Z)/(C*(D+Z)), LOWER BOUND = (B-A*Z)/(C*(D-Z))
(Z VALUES ARE STANDARD NORMAL VALUES FOR ALPHA)

SAMPLE	ALPHA = 0.90		ALPHA = 0.95		ALPHA = 0.975		ALPHA = 0.99	
SIZE	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND
30	1.8927	-2.9485	2.1805	-3.7110	2.4207	-4.4646	2.6915	-5.4811
40	1.9313	-2.8233	2.2337	-3.5193	2.4879	-4.1943	2.7764	-5.0849
50	1.9592	-2.7454	2.2721	-3.4017	2.5366	-4.0307	2.8382	-4.8494
60	1.9806	-2.6915	2.3017	-3.3209	2.5741	-3.9191	2.8860	-4.6907
61	1.9825	-2.6869	2.3043	-3.3141	2.5774	-3.9099	2.8902	-4.6776
62	1.9843	-2.6825	2.3068	-3.3075	2.5807	-3.9008	2.8943	-4.6649
63	1.9861	-2.6783	2.3093	-3.3012	2.5838	-3.8921	2.8984	-4.6525
64	1.9879	-2.6741	2.3118	-3.2950	2.5869	-3.8836	2.9023	-4.6405
65	1.9896	-2.6701	2.3142	-3.2890	2.5900	-3.8754	2.9062	-4.6289
66	1.9913	-2.6661	2.3165	-3.2831	2.5929	-3.8673	2.9100	-4.6176
67	1.9930	-2.6623	2.3188	-3.2774	2.5959	-3.8595	2.9137	-4.6066
68	1.9946	-2.6586	2.3211	-3.2719	2.5987	-3.8519	2.9174	-4.5959
69	1.9962	-2.6550	2.3233	-3.2665	2.6015	-3.8445	2.9210	-4.5855
70	1.9977	-2.6514	2.3254	-3.2612	2.6043	-3.8373	2.9245	-4.5753
71	1.9993	-2.6480	2.3276	-3.2561	2.6070	-3.8303	2.9279	-4.5655
72	2.0008	-2.6446	2.3297	-3.2511	2.6097	-3.8235	2.9313	-4.5559
73	2.0023	-2.6413	2.3317	-3.2462	2.6123	-3.8168	2.9347	-4.5465
74	2.0037	-2.6381	2.3337	-3.2414	2.6148	-3.8103	2.9379	-4.5373
75	2.0051	-2.6349	2.3357	-3.2368	2.6173	-3.8039	2.9412	-4.5284
76	2.0065	-2.6319	2.3376	-3.2322	2.6198	-3.7977	2.9443	-4.5197
77	2.0079	-2.6289	2.3395	-3.2278	2.6222	-3.7917	2.9474	-4.5112
78	2.0093	-2.6259	2.3414	-3.2234	2.6246	-3.7857	2.9505	-4.5029
79	2.0106	-2.6231	2.3433	-3.2192	2.6270	-3.7800	2.9535	-4.4948
80	2.0119	-2.6203	2.3451	-3.2150	2.6293	-3.7743	2.9564	-4.4869
81	2.0132	-2.6175	2.3469	-3.2110	2.6316	-3.7688	2.9594	-4.4792
82	2.0145	-2.6148	2.3486	-3.2070	2.6339	-3.7633	2.9622	-4.4716
83	2.0157	-2.6122	2.3503	-3.2031	2.6360	-3.7580	2.9650	-4.4642
84	2.0169	-2.6096	2.3520	-3.1993	2.6382	-3.7529	2.9678	-4.4570
85	2.0181	-2.6071	2.3537	-3.1956	2.6403	-3.7478	2.9705	-4.4499
86	2.0193	-2.6046	2.3554	-3.1919	2.6424	-3.7428	2.9732	-4.4430
87	2.0205	-2.6022	2.3570	-3.1883	2.6445	-3.7379	2.9759	-4.4362
88	2.0216	-2.5998	2.3586	-3.1848	2.6465	-3.7332	2.9785	-4.4295
89	2.0228	-2.5975	2.3601	-3.1814	2.6485	-3.7285	2.9810	-4.4230
90	2.0239	-2.5952	2.3617	-3.1780	2.6505	-3.7239	2.9836	-4.4166
91	2.0250	-2.5930	2.3632	-3.1747	2.6524	-3.7194	2.9861	-4.4103
92	2.0261	-2.5908	2.3647	-3.1714	2.6543	-3.7150	2.9885	-4.4042
93	2.0271	-2.5886	2.3662	-3.1683	2.6562	-3.7107	2.9909	-4.3982
94	2.0282	-2.5865	2.3677	-3.1651	2.6581	-3.7064	2.9933	-4.3923
95	2.0292	-2.5844	2.3691	-3.1621	2.6599	-3.7023	2.9957	-4.3865
100	2.0342	-2.5745	2.3761	-3.1475	2.6687	-3.6825	3.0070	-4.3590
120	2.0512	-2.5421	2.3996	-3.0999	2.6988	-3.6181	3.0456	-4.2699

TABLE 13.-CONTINUED

SAMPLE SIZE	ALPHA = 0.90 UPPER BOUND	ALPHA = 0.90 LOWER BOUND	ALPHA = 0.95 UPPER BOUND	ALPHA = 0.95 LOWER BOUND	ALPHA = 0.975 UPPER BOUND	ALPHA = 0.975 LOWER BOUND	ALPHA = 0.99 UPPER BOUND	ALPHA = 0.99 LOWER BOUND
140	2.0681	-2.5222	2.4225	-3.0696	2.7275	-3.5763	3.0818	-4.2109
160	2.0788	-2.5023	2.4374	-3.0417	2.7467	-3.5376	3.1066	-4.1580
180	2.0878	-2.4862	2.4500	-3.0174	2.7629	-3.5064	3.1276	-4.1153
200	2.0955	-2.4728	2.4608	-2.9980	2.7768	-3.4805	3.1457	-4.0801
220	2.1022	-2.4615	2.4702	-2.9816	2.7890	-3.4586	3.1614	-4.0503
240	2.1081	-2.4517	2.4786	-2.9674	2.7997	-3.4398	3.1754	-4.0247
260	2.1134	-2.4432	2.4860	-2.9551	2.8093	-3.4234	3.1878	-4.0025
280	2.1181	-2.4356	2.4926	-2.9443	2.8179	-3.4090	3.1989	-3.9830
300	2.1224	-2.4289	2.4987	-2.9346	2.8256	-3.3961	3.2091	-3.9656
320	2.1263	-2.4229	2.5041	-2.9259	2.8327	-3.3846	3.2183	-3.9500
340	2.1299	-2.4175	2.5092	-2.9181	2.8392	-3.3742	3.2268	-3.9360
360	2.1332	-2.4125	2.5138	-2.9110	2.8452	-3.3648	3.2346	-3.9232
380	2.1362	-2.4080	2.5181	-2.9045	2.8508	-3.3562	3.2418	-3.9116
400	2.1391	-2.4039	2.5221	-2.8985	2.8559	-3.3483	3.2485	-3.9009
420	2.1417	-2.4000	2.5258	-2.8930	2.8607	-3.3409	3.2547	-3.8911
440	2.1442	-2.3965	2.5293	-2.8879	2.8652	-3.3342	3.2606	-3.8820
460	2.1465	-2.3932	2.5325	-2.8831	2.8694	-3.3279	3.2661	-3.8735
480	2.1487	-2.3901	2.5356	-2.8787	2.8734	-3.3220	3.2713	-3.8656
500	2.1507	-2.3872	2.5385	-2.8745	2.8771	-3.3165	3.2761	-3.8582
520	2.1527	-2.3845	2.5412	-2.8707	2.8806	-3.3114	3.2807	-3.8513
540	2.1545	-2.3819	2.5438	-2.8670	2.8840	-3.3066	3.2851	-3.8448
560	2.1562	-2.3795	2.5462	-2.8635	2.8872	-3.3020	3.2892	-3.8387
580	2.1579	-2.3772	2.5486	-2.8603	2.8902	-3.2977	3.2932	-3.8329
600	2.1595	-2.3751	2.5508	-2.8572	2.8930	-3.2936	3.2969	-3.8274
620	2.1610	-2.3731	2.5529	-2.8543	2.8958	-3.2898	3.3005	-3.8222
640	2.1624	-2.3711	2.5549	-2.8515	2.8984	-3.2861	3.3039	-3.8173
660	2.1638	-2.3693	2.5569	-2.8489	2.9009	-3.2826	3.3072	-3.8126
680	2.1651	-2.3675	2.5587	-2.8463	2.9033	-3.2793	3.3103	-3.8081
700	2.1663	-2.3658	2.5605	-2.8439	2.9056	-3.2761	3.3133	-3.8039
720	2.1675	-2.3642	2.5622	-2.8416	2.9078	-3.2731	3.3162	-3.7998
740	2.1687	-2.3627	2.5638	-2.8394	2.9099	-3.2702	3.3190	-3.7959
760	2.1698	-2.3612	2.5654	-2.8373	2.9120	-3.2674	3.3217	-3.7922
780	2.1709	-2.3598	2.5669	-2.8353	2.9139	-3.2647	3.3242	-3.7886
800	2.1719	-2.3584	2.5684	-2.8333	2.9158	-3.2621	3.3267	-3.7852
820	2.1729	-2.3571	2.5698	-2.8315	2.9177	-3.2597	3.3291	-3.7819
840	2.1739	-2.3559	2.5712	-2.8297	2.9194	-3.2573	3.3314	-3.7787
860	2.1748	-2.3547	2.5725	-2.8279	2.9211	-3.2550	3.3337	-3.7756
880	2.1757	-2.3535	2.5738	-2.8263	2.9228	-3.2528	3.3358	-3.7727
900	2.1766	-2.3524	2.5750	-2.8247	2.9244	-3.2507	3.3379	-3.7693
920	2.1775	-2.3513	2.5762	-2.8231	2.9259	-3.2486	3.3399	-3.7671
940	2.1783	-2.3502	2.5773	-2.8216	2.9274	-3.2467	3.3419	-3.7644

TABLE 14. UPPER AND LOWER CONFIDENCE BOUNDS OF T =(MU HAT - MU)/S(MU)
FOR ALPHA =0.90, 0.95, 0.975, AND 0.99
P=.99
UPPER BOUND = (B+A*Z)/(C*(D+Z)), LOWER BOUND = (B-A*Z)/(C*(D-Z))
(Z VALUES ARE STANDARD NORMAL VALUES FOR ALPHA)

SAMPLE	ALPHA = 0.90		ALPHA = 0.95		ALPHA = 0.975		ALPHA = 0.99	
SIZE	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND
30	1.8626	-2.9017	2.1458	-3.6520	2.3823	-4.3937	2.6488	-5.3940
40	1.9008	-2.7788	2.1984	-3.4638	2.4487	-4.1282	2.7326	-5.0047
50	1.9284	-2.7023	2.2364	-3.3483	2.4967	-3.9674	2.7936	-4.7733
60	1.9496	-2.6493	2.2657	-3.2689	2.5338	-3.8578	2.8408	-4.6173
61	1.9514	-2.6449	2.2682	-3.2623	2.5371	-3.8487	2.8450	-4.6044
62	1.9533	-2.6406	2.2707	-3.2558	2.5403	-3.8398	2.8490	-4.5919
63	1.9550	-2.6364	2.2732	-3.2496	2.5434	-3.8313	2.8530	-4.5798
64	1.9568	-2.6323	2.2756	-3.2435	2.5465	-3.8229	2.8569	-4.5680
65	1.9585	-2.6283	2.2780	-3.2376	2.5495	-3.8148	2.8608	-4.5565
66	1.9602	-2.6245	2.2803	-3.2318	2.5524	-3.8069	2.8645	-4.5454
67	1.9618	-2.6207	2.2826	-3.2262	2.5553	-3.7992	2.8682	-4.5346
68	1.9634	-2.6171	2.2848	-3.2208	2.5581	-3.7918	2.8718	-4.5241
69	1.9650	-2.6135	2.2871	-3.2155	2.5609	-3.7845	2.8753	-4.5139
70	1.9665	-2.6100	2.2891	-3.2103	2.5635	-3.7774	2.8788	-4.5039
71	1.9681	-2.6066	2.2912	-3.2053	2.5663	-3.7705	2.8822	-4.4942
72	1.9695	-2.6033	2.2933	-3.2003	2.5689	-3.7638	2.8856	-4.4848
73	1.9710	-2.6001	2.2953	-3.1955	2.5715	-3.7572	2.8889	-4.4755
74	1.9724	-2.5969	2.2973	-3.1909	2.5740	-3.7508	2.8921	-4.4666
75	1.9739	-2.5938	2.2992	-3.1863	2.5765	-3.7446	2.8953	-4.4578
76	1.9752	-2.5908	2.3012	-3.1818	2.5789	-3.7385	2.8984	-4.4492
77	1.9766	-2.5879	2.3031	-3.1774	2.5813	-3.7325	2.9015	-4.4409
78	1.9779	-2.5850	2.3049	-3.1732	2.5837	-3.7267	2.9045	-4.4327
79	1.9793	-2.5822	2.3067	-3.1690	2.5860	-3.7210	2.9074	-4.4248
80	1.9805	-2.5794	2.3085	-3.1649	2.5883	-3.7155	2.9104	-4.4170
81	1.9818	-2.5767	2.3103	-3.1609	2.5905	-3.7100	2.9132	-4.4094
82	1.9831	-2.5741	2.3120	-3.1570	2.5928	-3.7047	2.9160	-4.4019
83	1.9843	-2.5715	2.3137	-3.1532	2.5949	-3.6995	2.9188	-4.3947
84	1.9855	-2.5690	2.3154	-3.1495	2.5971	-3.6944	2.9216	-4.3875
85	1.9867	-2.5665	2.3170	-3.1458	2.5992	-3.6894	2.9243	-4.3806
86	1.9879	-2.5641	2.3187	-3.1422	2.6012	-3.6845	2.9269	-4.3738
87	1.9890	-2.5617	2.3203	-3.1387	2.6033	-3.6797	2.9295	-4.3671
88	1.9902	-2.5593	2.3218	-3.1352	2.6053	-3.6750	2.9321	-4.3605
89	1.9913	-2.5571	2.3234	-3.1319	2.6073	-3.6704	2.9346	-4.3541
90	1.9924	-2.5548	2.3249	-3.1285	2.6092	-3.6659	2.9371	-4.3478
91	1.9935	-2.5526	2.3264	-3.1253	2.6111	-3.6615	2.9396	-4.3417
92	1.9945	-2.5504	2.3279	-3.1221	2.6130	-3.6572	2.9420	-4.3356
93	1.9956	-2.5483	2.3294	-3.1190	2.6149	-3.6529	2.9444	-4.3297
94	1.9966	-2.5462	2.3308	-3.1159	2.6167	-3.6487	2.9467	-4.3239
95	1.9977	-2.5442	2.3322	-3.1129	2.6185	-3.6446	2.9491	-4.3182
100	2.0026	-2.5345	2.3391	-3.0985	2.6272	-3.6252	2.9602	-4.2913
120	2.0193	-2.5026	2.3623	-3.0517	2.6569	-3.5619	2.9984	-4.2036

TABLE 14.-CONTINUED

SAMPLE SIZE	ALPHA = 0.90 UPPER BOUND	ALPHA = 0.90 LOWER BOUND	ALPHA = 0.95 UPPER BOUND	ALPHA = 0.95 LOWER BOUND	ALPHA = 0.975 UPPER BOUND	ALPHA = 0.975 LOWER BOUND	ALPHA = 0.99 UPPER BOUND	ALPHA = 0.99 LOWER BOUND
140	2.0363	-2.4833	2.3852	-3.0223	2.6855	-3.5212	3.0344	-4.1461
160	2.0468	-2.4638	2.3999	-2.9939	2.7044	-3.4831	3.0588	-4.0940
180	2.0556	-2.4479	2.4123	-2.9709	2.7204	-3.4524	3.0795	-4.0520
200	2.0632	-2.4347	2.4229	-2.9518	2.7341	-3.4269	3.0972	-4.0172
220	2.0698	-2.4236	2.4322	-2.9357	2.7460	-3.4053	3.1128	-3.9879
240	2.0756	-2.4139	2.4404	-2.9217	2.7566	-3.3868	3.1265	-3.9628
260	2.0808	-2.4055	2.4477	-2.9096	2.7660	-3.3707	3.1387	-3.9409
280	2.0855	-2.3981	2.4542	-2.8989	2.7745	-3.3565	3.1497	-3.9216
300	2.0897	-2.3915	2.4602	-2.8894	2.7821	-3.3438	3.1596	-3.9045
320	2.0936	-2.3856	2.4656	-2.8809	2.7891	-3.3325	3.1687	-3.8892
340	2.0971	-2.3802	2.4705	-2.8732	2.7955	-3.3223	3.1771	-3.8754
360	2.1003	-2.3754	2.4751	-2.8662	2.8014	-3.3130	3.1847	-3.8628
380	2.1033	-2.3709	2.4793	-2.8597	2.8069	-3.3045	3.1918	-3.8514
400	2.1061	-2.3668	2.4832	-2.8539	2.8119	-3.2967	3.1984	-3.8408
420	2.1087	-2.3631	2.4869	-2.8484	2.8167	-3.2895	3.2046	-3.8311
440	2.1111	-2.3595	2.4903	-2.8434	2.8211	-3.2828	3.2104	-3.8222
460	2.1134	-2.3563	2.4935	-2.8387	2.8252	-3.2766	3.2158	-3.8138
480	2.1156	-2.3533	2.4965	-2.8344	2.8291	-3.2709	3.2209	-3.8061
500	2.1176	-2.3504	2.4994	-2.8303	2.8328	-3.2655	3.2257	-3.7988
520	2.1195	-2.3477	2.5021	-2.8264	2.8363	-3.2604	3.2302	-3.7920
540	2.1213	-2.3452	2.5046	-2.8228	2.8395	-3.2556	3.2345	-3.7856
560	2.1230	-2.3429	2.5070	-2.8194	2.8427	-3.2512	3.2386	-3.7795
580	2.1246	-2.3406	2.5093	-2.8162	2.8457	-3.2469	3.2425	-3.7739
600	2.1262	-2.3385	2.5115	-2.8132	2.8485	-3.2429	3.2461	-3.7685
620	2.1277	-2.3365	2.5136	-2.8103	2.8512	-3.2391	3.2497	-3.7634
640	2.1291	-2.3346	2.5156	-2.8076	2.8538	-3.2355	3.2530	-3.7585
660	2.1304	-2.3328	2.5175	-2.8050	2.8562	-3.2320	3.2563	-3.7539
680	2.1317	-2.3310	2.5193	-2.8025	2.8586	-3.2288	3.2593	-3.7495
700	2.1330	-2.3294	2.5211	-2.8001	2.8608	-3.2256	3.2623	-3.7453
720	2.1342	-2.3278	2.5227	-2.7979	2.8630	-3.2226	3.2651	-3.7413
740	2.1353	-2.3263	2.5243	-2.7957	2.8651	-3.2198	3.2679	-3.7375
760	2.1364	-2.3248	2.5259	-2.7936	2.8671	-3.2170	3.2705	-3.7338
780	2.1375	-2.3234	2.5274	-2.7916	2.8690	-3.2144	3.2730	-3.7303
800	2.1385	-2.3221	2.5288	-2.7897	2.8709	-3.2119	3.2755	-3.7269
820	2.1395	-2.3208	2.5302	-2.7879	2.8727	-3.2095	3.2778	-3.7236
840	2.1404	-2.3196	2.5316	-2.7861	2.8745	-3.2071	3.2801	-3.7205
860	2.1413	-2.3184	2.5329	-2.7844	2.8761	-3.2049	3.2823	-3.7175
880	2.1422	-2.3172	2.5341	-2.7827	2.8778	-3.2027	3.2844	-3.7146
900	2.1431	-2.3161	2.5353	-2.7812	2.8793	-3.2006	3.2865	-3.7118
920	2.1439	-2.3151	2.5365	-2.7796	2.8809	-3.1986	3.2885	-3.7091
940	2.1447	-2.3140	2.5376	-2.7781	2.8823	-3.1967	3.2904	-3.7065

TABLE 15. UPPER AND LOWER CONFIDENCE BOUNDS OF T =(MU HAT - MU)/S(MU)
FOR ALPHA =0.90, 0.95, 0.975, AND 0.99

p =.995
UPPER BOUND = (B+A*Z)/(C*(D+Z)), LOWER BOUND = (B-A*Z)/(C*(D-Z))
(Z VALUES ARE STANDARD NORMAL VALUES FOR ALPHA)

SAMPLE	ALPHA = 0.90		ALPHA = 0.95		ALPHA = 0.975		ALPHA = 0.99	
SIZE	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND	LOWER BOUND
30	1.8422	-2.8700	2.1224	-3.6122	2.3562	-4.3458	2.6198	-5.3351
40	1.8892	-2.7486	2.1746	-3.4262	2.4221	-4.0834	2.7030	-4.9504
50	1.9075	-2.6731	2.2122	-3.3121	2.4697	-3.9245	2.7634	-4.7216
60	1.9285	-2.6208	2.2412	-3.2336	2.5065	-3.8162	2.8101	-4.5675
61	1.9304	-2.6164	2.2438	-3.2271	2.5097	-3.8072	2.8143	-4.5548
62	1.9322	-2.6121	2.2462	-3.2207	2.5129	-3.7984	2.8183	-4.5424
63	1.9339	-2.6080	2.2487	-3.2145	2.5160	-3.7899	2.8222	-4.5304
64	1.9357	-2.6039	2.2511	-3.2085	2.5190	-3.7817	2.8261	-4.5187
65	1.9374	-2.6000	2.2534	-3.2027	2.5220	-3.7737	2.8299	-4.5074
66	1.9390	-2.5962	2.2557	-3.1970	2.5249	-3.7659	2.8336	-4.4964
67	1.9407	-2.5925	2.2580	-3.1915	2.5277	-3.7583	2.8373	-4.4857
68	1.9422	-2.5889	2.2602	-3.1861	2.5306	-3.7509	2.8408	-4.4753
69	1.9438	-2.5853	2.2623	-3.1808	2.5333	-3.7437	2.8444	-4.4652
70	1.9453	-2.5819	2.2645	-3.1757	2.5360	-3.7367	2.8478	-4.4554
71	1.9469	-2.5785	2.2665	-3.1707	2.5386	-3.7299	2.8512	-4.4458
72	1.9483	-2.5753	2.2686	-3.1659	2.5412	-3.7233	2.8545	-4.4364
73	1.9498	-2.5721	2.2706	-3.1611	2.5438	-3.7168	2.8577	-4.4273
74	1.9512	-2.5690	2.2726	-3.1565	2.5463	-3.7105	2.8609	-4.4185
75	1.9526	-2.5659	2.2745	-3.1520	2.5488	-3.7043	2.8641	-4.4098
76	1.9540	-2.5629	2.2764	-3.1476	2.5512	-3.6982	2.8672	-4.4013
77	1.9553	-2.5600	2.2783	-3.1432	2.5535	-3.6924	2.8702	-4.3931
78	1.9566	-2.5572	2.2801	-3.1390	2.5559	-3.6866	2.8732	-4.3850
79	1.9579	-2.5544	2.2819	-3.1349	2.5582	-3.6810	2.8761	-4.3772
80	1.9592	-2.5517	2.2837	-3.1309	2.5604	-3.6755	2.8790	-4.3695
81	1.9605	-2.5490	2.2854	-3.1269	2.5627	-3.6701	2.8819	-4.3619
82	1.9617	-2.5464	2.2871	-3.1231	2.5649	-3.6648	2.8847	-4.3546
83	1.9629	-2.5438	2.2888	-3.1193	2.5670	-3.6597	2.8874	-4.3474
84	1.9641	-2.5413	2.2905	-3.1156	2.5691	-3.6547	2.8901	-4.3404
85	1.9653	-2.5389	2.2921	-3.1120	2.5712	-3.6497	2.8928	-4.3335
86	1.9665	-2.5365	2.2937	-3.1084	2.5733	-3.6449	2.8954	-4.3267
87	1.9676	-2.5341	2.2953	-3.1049	2.5753	-3.6402	2.8980	-4.3201
88	1.9688	-2.5318	2.2969	-3.1015	2.5773	-3.6355	2.9006	-4.3136
89	1.9699	-2.5296	2.2984	-3.0982	2.5792	-3.6310	2.9031	-4.3073
90	1.9710	-2.5273	2.2999	-3.0949	2.5812	-3.6265	2.9055	-4.3011
91	1.9720	-2.5252	2.3014	-3.0917	2.5831	-3.6222	2.9080	-4.2950
92	1.9731	-2.5230	2.3029	-3.0885	2.5849	-3.6179	2.9104	-4.2890
93	1.9741	-2.5209	2.3043	-3.0854	2.5868	-3.6137	2.9127	-4.2832
94	1.9752	-2.5189	2.3058	-3.0824	2.5886	-3.6095	2.9151	-4.2774
95	1.9762	-2.5168	2.3072	-3.0794	2.5904	-3.6055	2.9174	-4.2718
100	1.9810	-2.5072	2.3139	-3.0653	2.5990	-3.5863	2.9284	-4.2452
120	1.9976	-2.4758	2.3370	-3.0190	2.6284	-3.5237	2.9662	-4.1585

TABLE 15.-CONTINUED

SAMPLE SIZE	ALPHA = 0.90 UPPER BOUND	ALPHA = 0.90 LOWER BOUND	ALPHA = 0.95 UPPER BOUND	ALPHA = 0.95 LOWER BOUND	ALPHA = 0.975 UPPER BOUND	ALPHA = 0.975 LOWER BOUND	ALPHA = 0.99 UPPER BOUND	ALPHA = 0.99 LOWER BOUND
140	2.0146	-2.4569	2.3597	-2.9901	2.6569	-3.4837	3.0020	-4.1019
160	2.0249	-2.4375	2.3743	-2.9620	2.6756	-3.4460	3.0262	-4.0503
180	2.0337	-2.4218	2.3865	-2.9392	2.6913	-3.4156	3.0466	-4.0088
200	2.0412	-2.4088	2.3971	-2.9204	2.7049	-3.3904	3.0642	-3.9744
220	2.0477	-2.3977	2.4063	-2.9044	2.7167	-3.3690	3.0796	-3.9454
240	2.0535	-2.3882	2.4144	-2.8906	2.7272	-3.3507	3.0931	-3.9205
260	2.0586	-2.3799	2.4216	-2.8786	2.7365	-3.3347	3.1052	-3.8989
280	2.0633	-2.3726	2.4281	-2.8680	2.7449	-3.3207	3.1161	-3.8798
300	2.0674	-2.3660	2.4339	-2.8586	2.7524	-3.3082	3.1259	-3.8629
320	2.0712	-2.3602	2.4393	-2.8502	2.7594	-3.2970	3.1349	-3.8477
340	2.0747	-2.3549	2.4442	-2.8425	2.7657	-3.2868	3.1432	-3.8340
360	2.0779	-2.3500	2.4487	-2.8356	2.7715	-3.2776	3.1508	-3.8216
380	2.0809	-2.3456	2.4529	-2.8292	2.7769	-3.2692	3.1578	-3.8103
400	2.0837	-2.3416	2.4568	-2.8234	2.7819	-3.2615	3.1643	-3.7999
420	2.0862	-2.3378	2.4604	-2.8180	2.7866	-3.2544	3.1704	-3.7903
440	2.0886	-2.3344	2.4638	-2.8131	2.7910	-3.2478	3.1761	-3.7814
460	2.0909	-2.3312	2.4669	-2.8084	2.7951	-3.2417	3.1815	-3.7731
480	2.0930	-2.3282	2.4699	-2.8041	2.7989	-3.2360	3.1865	-3.7655
500	2.0950	-2.3253	2.4727	-2.8001	2.8026	-3.2306	3.1913	-3.7583
520	2.0969	-2.3227	2.4754	-2.7963	2.8060	-3.2256	3.1957	-3.7515
540	2.0987	-2.3202	2.4779	-2.7927	2.8093	-3.2209	3.2000	-3.7452
560	2.1004	-2.3179	2.4803	-2.7894	2.8124	-3.2165	3.2040	-3.7392
580	2.1020	-2.3157	2.4826	-2.7862	2.8153	-3.2123	3.2079	-3.7336
600	2.1035	-2.3136	2.4847	-2.7832	2.8181	-3.2083	3.2115	-3.7283
620	2.1050	-2.3116	2.4868	-2.7803	2.8208	-3.2045	3.2150	-3.7232
640	2.1064	-2.3097	2.4887	-2.7776	2.8233	-3.2010	3.2183	-3.7184
660	2.1077	-2.3079	2.4906	-2.7751	2.8257	-3.1976	3.2215	-3.7138
680	2.1090	-2.3062	2.4924	-2.7726	2.8281	-3.1943	3.2246	-3.7095
700	2.1102	-2.3045	2.4942	-2.7703	2.8303	-3.1912	3.2275	-3.7053
720	2.1114	-2.3030	2.4958	-2.7680	2.8325	-3.1883	3.2303	-3.7014
740	2.1125	-2.3015	2.4974	-2.7659	2.8345	-3.1854	3.2330	-3.6976
760	2.1136	-2.3000	2.4989	-2.7638	2.8365	-3.1827	3.2356	-3.6940
780	2.1147	-2.2987	2.5004	-2.7618	2.8384	-3.1801	3.2381	-3.6905
800	2.1157	-2.2973	2.5018	-2.7599	2.8403	-3.1776	3.2405	-3.6871
820	2.1166	-2.2961	2.5032	-2.7581	2.8421	-3.1752	3.2429	-3.6839
840	2.1176	-2.2948	2.5046	-2.7564	2.8438	-3.1729	3.2451	-3.6808
860	2.1185	-2.2937	2.5058	-2.7547	2.8455	-3.1707	3.2473	-3.6778
880	2.1194	-2.2925	2.5071	-2.7531	2.8471	-3.1686	3.2494	-3.6750
900	2.1202	-2.2914	2.5083	-2.7515	2.8486	-3.1665	3.2514	-3.6722
920	2.1210	-2.2904	2.5094	-2.7500	2.8501	-3.1645	3.2534	-3.6695
940	2.1218	-2.2893	2.5106	-2.7485	2.8516	-3.1626	3.2553	-3.6669

APPENDIX B

This appendix presents the computer program used to compute the density functions for a specified extreme mean and its confidence intervals.

```
      PROGRAM MAIN(OUTPUT,TAPE13)
      COMMON /SHARE/ N(41),K,DEN(103,9),DIST(103,9),T(103),INDEX(9)
      COMMON /CHARE/ A(41),B(41),C(41),D(41),DD(41),F(41)
C****              ROUTINE TO PRODUCE TABLE FOR DISTRIBUTION OF
C****              VALUES ABOVE A..
C****
C****              CONFIDENCE INTERVAL COMPUTATIONS..
C****
      DATA N/30,40,50,60/
      AA = 1.753975
      DO 1 I=5,39
  1   N(I) = 60 + I - 4
C****
      N(40) = 100
      N(41) = 120
C****
      DO 7 L=1,2
      DO 2 J=1,3
      DO 4 I=1,41
      IF(N(I).LE.120)
     -F(I)=SQRT(FLOAT(N(I)-1)/2.)*GAMMA(FLOAT(N(I)-1)/2.)/GAMMA(FLOAT(N(
     -I))/2.)
      IF(N(I).GT.120)
     -F(I)=EXP(+.5)*(FLOAT(N(I)-1)/FLOAT(N(I)))**((FLOAT(N(I))-1.)/2.)
      A(I) = 1./SQRT(FLOAT(N(I))) + AA*F(I)/SQRT(FLOAT(2*N(I)-1))
      B(I) = AA*(F(I)*SQRT(FLOAT(2*N(I)-3)/FLOAT(2*N(I)-2))-1.)
      DD(I) = FLOAT(N(I))*AA*AA*(F(I)*F(I)-1.)
      C(I) = SQRT((1.+DD(I))/FLOAT(2*N(I)*N(I)-1))
    4 D(I) = SQRT(FLOAT(2*N(I)-3))
C****
      IF(L.EQ.2)3,6
C****              PRINT RESULTS, TABLE 1..
    6 IF(J.EQ.1)CALL PRINT1
C****              PRINT RESULTS, TABLE 2..
    3 IF(J.EQ.2)CALL PRINT2
      IF(J.EQ.3)PRINT 104
  104  FORMAT(" "76("-"))
  2   CONTINUE
C****
      CALL DRAW1
      N(1)=140
      DO 98 K=2,41
   98 N(K)=N(K-1)+20
C****
    7 K=0
C****
      CALL PLOT(0.,0.,999)
      END
```

```
      SUBROUTINE DRAW1
      DIMENSION ZNORM(103,9)
      COMMON /SHARE/ N(41),K,DEN(103,9),DIST(103,9),T(103),INDEX(9)
      COMMON /CHARE/ A(41),B(41),C(41),D(41),DD(41),F(41)
      DATA INDEX/1,2,3,4,14,24,34,40,41/
      DATA PI/17216220773250420551B/
      CALL PLOTS(0,0,13)
      CALL FACTOR(2./2.54)
      CALL PLOT(1.,2.,-3)
      T(102) = -5.
      T(103) = 1.
      ZNORM(102) = 0.
      ZNORM(103) = .05
C****                FILL ARRAYS WITH DENSITY AND DISTRIBUTION VALUES..
C****
      DO 109 K=1,9
      I = INDEX(K)
C****
      DO 3 J=1,101
      T(J) = -5. + FLOAT(J-1)/10.
      ZNORM(J) = 1./SQRT(2.*PI*EXP(T(J)*T(J)))
      X = (C(I)*D(I)*T(J) - B(I))/(A(I)-C(I)*T(J))
      DIST(J,K) = FNORMAL(X)
      DEN(J,K) = C(I)*(A(I)*D(I)-B(I))/(SQRT(2.*PI)*(A(I)-C(I)*T(J))**2)
     -*EXP(-(C(I)*D(I)*T(J)-B(I))**2/(2.*(A(I)-C(I)*T(J))**2))
    3 CONTINUE
C****
      CALL AXIS(0.,0.," ",-1,10.,0.,-5.,1.)
      CALL AX90(5.,0.," ",1,8.,90.,0.,.05)
      DEN(102,K) = 0.
      DEN(103,K) = .05
C****                DO THE PLOTTING STUFF ....
      CALL LINE(T,DEN(1,K),101,1,0,0)
      CALL DASHL(T,ZNORM,101,1,0,0)
      CALL SYMBOL(1.,8.5,.15,"DENSITY FUNCTION OF Z(...) AND T(-) = (MU H
     -HAT - MU)/S(MU HAT)",0.,62)
      CALL SYMBOL(8.5,8.,.15,"N = ",0.,4)
      CALL NUMBER(9.1,8.,.15,FLOAT(N(INDEX(K))),0.,-1)
      CALL PLOT(12.,0.,-3)
C****                PRINT RESULTS, TABLE 3..
  109 CALL PRINT4
C****
      RETURN
      END
```

```
      SUBROUTINE PRINT1
      DIMENSION Z(4)
      COMMON /SHARE/ N(41),K,DEN(103,9),DIST(103,9),T(103),INDEX(9)
      COMMON /CHARE/ A(41),B(41),C(41),D(41),DD(41),F(41)
      FCNU(A,B,C,D,Z) = (B+A*Z)/(C*(D+Z))
      FCNL(A,B,C,D,Z) = (B-A*Z)/(C*(D-Z))
      DATA Z/1.645,1.96, 2.24,2.576/
      PRINT 100,(N(I),F(I),SQRT((1.+DD(I))/FLOAT(N(I))),A(I),B(I),C(I),
     -D(I),I=1,41)
100   FORMAT(*1*,//,*  TABLE 1.*,/,T10,"VARIOUS FUNCTION OF N NEEDED IN
     -THE CALCULATION OF THE",/,T10,"DISTRIBUTION OF T = (MU HAT - MU)/S
     -(MU HAT)",//,T15,"F = (GAMMA((N-1)/2)/GAMMA(N/2))*SQRT((N-1)/2)",/
     -,              T15,"D1 = N*(F*F - 1)*A1*A1",/,
     -               T15,"A = 1./SQRT(N) + A1*F*(1./SQRT(2*(N-1)))",/,
     -               T15,"B = A1*(F*SQRT( 2*N-3)/(2*N-2) )  -1)",/,
     -               T15,"C = SQRT( (1 + D1)/(2*N*(N-1) )",/,
     -               T15,"D = SQRT(2*N-3)",////,
     -"  N",T13,"F",T20,"SQRT((1+D1)/N)",T38,"A",T48,"B",T58,"C",T68,"D"
     -,//,
     -(" "I3,T8,F9.6,T20,F9.6,T33,F9.6,T43,F9.6,T53,F9.6,T63,F9.6))
      RETURN
      ENTRY PRINT2
    4 PRINT 102,(N(I),(FCNU(A(I),B(I),C(I),D(I),Z(M)),
     -               FCNL(A(I),B(I),C(I),D(I),Z(M)),M=1,4),I=1,41)
  102 FORMAT("1",//"  TABLE 2: UPPER AND LOWER CONFIDENCE BOUNDS OF T =
     -(MU HAT - MU)/S(MU)"/T15,"FOR ALPHA =0.90, 0.95, 0.975, AND 0.99"//
     - T12,"C=.9"
     -/T12,"UPPER BOUND = (B+A*Z)/(C*(D+Z)), LOWER BOUND = (B-A*Z)/(C*(D
     --Z))"/,T15,"(Z VALUES ARE STANDARD NORMAL VALUES FOR ALPHA)"//" ",
     -76("-"),/,T2,"SAMPLE",T9,"I ALPHA = 0.90",T26,"I ALPHA = 0.95",T43
     -,"I ALPHA = 0.975",T60,"I ALPHA = 0.99",T77,"I",/,T9,69("-"),/,
     -T4,"SIZE",T9,"I UPPER I  LOWER",T26,"I UPPER I  LOWER",T43,"I UPPE
     -R I  LOWER",T60,"I UPPER I  LOWER",T77,"I",/,T9,
     -"I BOUND I  BOUND",T26,"I BOUND I  BOUND",T43,"I BOUND I  BOUND",
     -T60,"I BOUND I  BOUND",T77,"I",/,T2,76("-"),/
     -       (" "I3,T9,"I ",F6.4,1X,F7.4,T26,"I ",F6.4,1X,F7.4,T43,"I ",
     -F6.4,1X,F7.4,T60,"I ",F6.4,1X,F7.4,T77,"I"))
      RETURN
      ENTRY PRINT4
      PRINT 106, N(INDEX(K)),(T(II),DEN(II,K),DIST(II,K),T(II+50),DEN(II
     -+50,K),DIST(II+50,K),II=1,50)
  106 FORMAT(*1*,//,"  TABLE 3.  DENSITY ( D(T) ) AND DISTRIBUTION ( F(T
     -) ) OF THE RANDOM VARIABLE"/,T15,"T = (MU HAT - MU)/S(MU HAT)
     -N = ",I3,//,
     -T12,"T",T22,"D(T)",T32,"F(T)",T45,"T",T55,"D(T)",T65,"F(T)",/,
     -( 10(* *,T10,F6.2,T20,F9.6,T30,F9.6,T43,F6.2,T53,F9.6,T63,F9.6,/)
     -))
      RETURN
      END
```

```
      SUBROUTINE CINTRVL(XBAR,SIGMA,FCNU,FCNL,N,XLOW,XHIGH)
C****
C****                SUBROUTINE TO PRODUCE 95 PERCENT CONFIDENCE INTERVALS
C****                FOR EXTREEM MEAN
C****
C****                BY BROWNLOW, SDC/ISI  3/79
C****
C****                INPUT:
C****                  XBAR  DATA MEAN
C****                  SIGMA  DATA STANDARD DEVIATION
C****                  FCNU,FCNL  FUNCTIONAL VALUES PASSED IN FROM
C****                  N  NUMBER OF OBSERVATIONS IN THE SAMPLE..
C****                  MAIN ROUTINE WRITTEN BY CRUM.
C****
C****                OUTPUT:
C****                  XLOW  LOWER CONFIDENCE INTERVAL VALUE
C****                  XHIGH  UPPER CONFIDENCE INTERVAL VALUE.
C****
C****
C****                FOR VARIOUS PROBABILITY VALUES, AA AND Z MUST
C****                BE CHANGED..
C****                  SEE PAPER FOR DETAILS...
C****
      AA = 2.0627128034
      Z = 1.644853628
C****
C****
      FN = N
      F = EXP(.5)*((FN-1.)/FN)**( (FN-1.)/2.)
C****
      DD = FN*AA*(F*F-1.)
      XLOW = XBAR + AA*F*SIGMA - FCNU*SIGMA*SQRT((1.+DD)/FN)
      XHIGH = XBAR + AA*F*SIGMA - FCNL*SIGMA*SQRT((1.+DD)/FN)
C****
C****
C****
      RETURN
      END
```

REFERENCES

1. Bauer, Carol A.; Mackall, Karen G.; Stoll, Frederick; and Tremback, Jeffrey W.: Comparison of Flight and Wind Tunnel Model Instantaneous Distortion Data From a Mixed-Compression Inlet. YF-12 Experiments Symposium, Vol. 3. NASA CP-2054, 1978, pp. 295-375.

2. Fisz, Marek: Probability Theory and Mathematical Statistics. Third ed. John Wiley & Sons, Inc., c. 1963.

3. Zacks, Shelemyahu: The Theory of Statistical Inference. John Wiley & Sons, Inc., c. 1971.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-81346		

4. Title and Subtitle	5. Report Date
EXTREME MEAN AND ITS APPLICATIONS	December 1979
	6. Performing Organization Code
7. Author(s) Ram Swaroop and James D. Brownlow (System Development Corporation) and William R. Winter (NASA Dryden Flight Research Center)	8. Performing Organization Report No. H-1088
9. Performing Organization Name and Address NASA Dryden Flight Research Center P. O. Box 273 Edwards, California 93523	10. Work Unit No. 168-02-05
	11. Contract or Grant No. NAS4-2334
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546	13. Type of Report and Period Covered Technical Memorandum
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

In many applications, a number or results are predicated upon extreme value statistics obtained from normally distributed data. An extreme mean in this study is defined as the mean of p-th probability truncated normal distribution.

An unbiased estimate of this extreme mean and its large sample distribution are derived. The distribution of this estimate even for very large samples is found to be non-normal. Further, as the sample size increases, the variance of the unbiased estimate converges to the Cramer-Rao lower bound.

The computer program used to obtain the density and distribution functions of the standardized unbiased estimate, and the confidence intervals of the extreme mean for any data are included for ready application. An example is included to demonstrate the usefulness of extreme mean application.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Maximum value prediction Truncated distribution Normal distribution Unbiased estimate	Unclassified—Unlimited STAR category: 65

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	39	$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161